

${}^6\text{He}$ β -decay rate and the suppression of the axial constant in nuclear matter

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We present a microscopic calculation of the ${}^6\text{He}$ β -decay into the ground state of ${}^6\text{Li}$. To this end, we use chiral perturbation theory at next-to-next-to-next-to-leading order to describe the nuclear weak-currents. The nuclear wave functions are derived from the J-matrix inverse scattering nucleon-nucleon potential (JISP), and the Schrödinger equation is solved using the hyperspherical-harmonics expansion. Our calculation brings the theoretical decay-rate within 3% of the measured one. This success is attributed to the use of chiral-perturbation-theory based mesonic currents, whose contribution is qualitatively different compared to standard nuclear physics approach, where the use of meson exchange currents worsens the comparison to experiment. The inherent inconsistency in the use of the JISP potential together with chiral-perturbation-theory based is argued not to affect this conclusion, though a more detailed investigation is called for. We conclude that any suppression of the axial constant in nuclear matter is included in this description of the weak interaction in the nucleus.

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I. INTRODUCTION

β -decay is the every-day reflection of weak-interaction in nuclei. As such, it provides an experimental window to the properties of the weak interaction at nuclear density.

In particular, theoretical studies of β -decay rates of nuclei have argued for a suppression of the axial coupling constant g_A , from its vacuum value, as extracted from the lifetime of the neutron $g_A = 1.2695 \pm 0.0029$ [1], to unity, i.e. $g_A = 1$ [2, 3]. According to a recent study, this suppression occurs gradually, as the mass of the nucleus grows, and fully utilized for $A \approx 40$ [4].

The ramifications of this suppression are numerous. For example, to the understanding of astrophysical phenomena, such as neutron-star cooling and core collapse supernovae, whose dynamics is controlled by the weak interactions. It is of no surprise that the source of this suppression has been the target of many theoretical works, which have associated it with a partial restoration of chiral symmetry in finite densities, deficiencies in the inclusion of correlations between nucleons, loop-corrections to the axial current originating in nucleonic excitations and mesonic currents, or combination of the three [5, 6, 7, 8, 9].

An important assumption is hidden in these suppression mechanism: if a full calculation of the weak interaction inside nuclei was possible from first principles, then the calculated decay-rates should agree with the experi-

mental ones. That is, if one could describe correctly the correlations between nucleons, and the weak interaction of an external probe with a nucleus, then one should recover the physical value of the axial constant. In order to do that, pertinent is to solve the nuclear problem from first-principles. Due to the strong correlations involved in the problem, a calculation of nuclear wave functions from the nucleonic degrees of freedom is at reach only for very light nuclei.

The lightest nucleus that undergoes a β -decay is the triton. However, the theory cannot be checked in the triton since its half-life is used to remove some freedom in the weak interaction of a lepton with a nucleus, as will be explained explicitly later. The lightest nucleus that can provide a test to the theory is thus ${}^6\text{He}$. ${}^6\text{He}$ ($J^\pi = 0^+$) is an unstable nucleus, which undergoes a β decay with a half-life $\tau_{1/2} = 806.7 \pm 1.5$ msec to the ground state of ${}^6\text{Li}$ ($J^\pi = 1^+$) [10].

However, a microscopic calculation of ${}^6\text{He}$ from its nucleonic degrees of freedom, failed to reproduce the β -decay rate. This study, accomplished by Schiavilla and Wiringa [11], has used the realistic Argonne $v18$ (AV18) nucleon-nucleon potential, combined with the Urbana-IX (UIX) three-nucleon-force (3NF), to derive the nuclear wave functions, through the variational Monte-Carlo approach. The model used for the nuclear weak axial current includes one- and two-body operators. The two-body currents are phenomenological, with the strength of the leading two-body term – associated with Δ -isobar excitation of the nucleon – adjusted to reproduce the Gamow-Teller matrix element in tritium β -decay. The calculated half-life of ${}^6\text{He}$ overpredicts the measured one by about 9%. An unexpected result of the calculation, was that two-body currents lead to a 1.7% increase in

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the value of the Gamow-Teller matrix element of ${}^6\text{He}$, thus worsening the comparison with experiment. The authors of this paper have presumed that the origin of this discrepancy is either in the approximate character of the VMC wave functions, or in the discrepancies of the nuclear model of the weak interaction. Pervin *et al.* [12], have used the GFMC approach to evolve the VMC wave functions ansatz. They showed that this brings the single nucleon Gamow-Teller matrix element to about 0-3% deviation from the experimental value. However, the MEC are still expected to increase the deviation from the experiment to about 2-5% from the experimental value, leaving this problem intact.

In the current *paper*, we argue that the origin of the discrepancy is indeed in the model of the weak interaction inside the nucleus. The foundation of such an argument has to be in the underlying theory, i.e. quantum chromodynamics (QCD). Thus, we describe the weak currents within the nucleus, using an effective theory of QCD, namely chiral perturbation theory (χ PT), applicable at low energies, relevant to β -decay processes [13, 14, 15, 16]. We use the triton β -decay to calibrate the strength of the contact interaction part of the meson-exchange currents, thus the calculation is without any free-parameters. The six-body nuclear problem is solved in a fully *ab-initio* approach, expanded in hyperspherical-harmonics function, from its nucleonic degrees of freedom [17, 18, 19]. The nuclear wave functions are derived from J-matrix inverse scattering nucleon-nucleon potential (JISP), describing two-nucleon scattering data and bound and resonant states of light nuclei to high accuracy [20, 21, 22]. Using this approach not only brings the calculated β -decay rate to within 3% of the measured data, but also changes qualitatively the contribution of the two-body meson exchange currents (MEC) compared to the work of Schiavilla and Wiringa. χ PT based MEC are found to decrease the Gamow-Teller matrix element, compared with the increase found by Schiavilla and Wiringa. We argue that this qualitatively different behavior originates in the use of χ PT based MEC, rather than the specific choice of the potential.

II. THEORETICAL FORMALISM

We start with a brief reminder of β -decay process, and the formalism used in the calculation. The decay is a weak process, in which an unstable nucleus of charge Z emits an electron and anti-electron-neutrino, leaving a nucleus of charge $Z + 1$. The interaction is mediated through the exchange of heavy W^+ boson. As the momentum transfer in the process is much smaller than the mass of the W^+ boson, the weak interaction Hamiltonian is given by $\hat{H}_W = -\frac{G|V_{ud}|}{\sqrt{2}} \int d^3x \hat{j}_\mu^-(\vec{x}) \hat{J}^{+\mu}(\vec{x})$, where $G = 1.166371(6) \times 10^{-11} \text{ MeV}^{-2}$ is the Fermi coupling constant [1], $V_{ud} = 0.9738(4)$ is the CKM matrix element mixing u and d quarks involved in the process [1],

$\hat{j}_\mu^-(\vec{x})$ is the lepton charge lowering current, and $\hat{J}^{+\mu}$ is the nuclear charge raising current. The decay rate can be calculated using Fermi's Golden rule, and it is proportional to the squared matrix element of this weak Hamiltonian $\langle f | \hat{H}_W | i \rangle$, where i (f) is the initial (final) state. The lepton current is well-approximated as a current of charged Dirac particles, thus results in kinematical factors to the decay-rate. The weak nuclear current can be written as: $\hat{J}^{+\mu} = \frac{\tau_+}{2} (\hat{J}^{V\mu} + \hat{J}^{A\mu})$, where τ_+ is a Pauli matrix. $\hat{J}^{V\mu}$ ($\hat{J}^{A\mu}$) has a polar- (axial-) vector symmetry. Here, we will discuss either a triton decay, or ${}^6\text{He}$ decay, hence the transitions are constrained by a selection rule on the angular-momentum change in the transition: $\Delta J = 0, 1$. Thus, a multipole decomposition of the nuclear current is helpful. Due to the small momentum-transfer only the lowest multipoles contribute, i.e. the $J = 1$ electric multipole of axial-vector symmetry E_1^A , and in the case of triton also the $J = 0$ coulomb multipole of polar-vector symmetry C_0^V . We explicitly checked that indeed the contribution of neglected multipoles to the decay-rate of ${}^6\text{He}$ can be bounded by 1% [11, 23]. The leading order contribution to the E_1^A and C_0^V operators are proportional to the Gamow-Teller and Fermi operators, respectively. Thus, it is customary, when discussing the experimental rates, to talk about the empirical Fermi and Gamow-Teller matrix elements, instead of E_1^A and C_0^V , using the relations

$$F \equiv \sqrt{\frac{4\pi}{2J_i + 1}} \langle C_0^V \rangle, \quad (1)$$

and

$$\text{GT} \equiv \sqrt{\frac{6\pi}{2J_i + 1}} \frac{\langle E_1^A \rangle}{g_A}. \quad (2)$$

Here, $\langle C_0^V \rangle \equiv \langle f | C_0^V | i \rangle$, and similarly for $\langle E_1^A \rangle$, J_i is the total angular momentum of the initial nucleus, and $g_A = 1.2695 \pm 0.0029$ is the axial constant [1].

As discussed by Simpson [24], and later revisited by Schiavilla *et al.* [11, 25], the ‘‘comparative’’ half-life is related to the ‘‘empirical’’ GT and F operators thorough

$$(fT_{1/2})_t = \frac{K/(G^2|V_{ud}|^2)}{|F|^2 + \frac{f_A}{f_V} g_A^2 |\text{GT}|^2}. \quad (3)$$

Here, $K = 2\pi^3 \ln 2 / m_e^5$ (such that $K/(G^2|V_{ud}|^2) = 6146.6 \pm 0.6 \text{ sec}$), and $f_A/f_V = 1.00529$ [25] accounts for the small difference in the statistical rate function between vector and axial-vector transitions. Putting the measured ${}^6\text{He}$ comparative half-life $(fT_{1/2})_t = 812.8 \pm 3.7 \text{ sec}$ [11], one extracts $|\text{GT}({}^6\text{He})|_{\text{expt}} = 2.161 \pm 0.005$. For triton, $(fT_{1/2})_t = 1129.6 \pm 3 \text{ sec}$ [26], thus $|\text{GT}({}^3\text{H})|_{\text{expt}} = 1.6560 \pm 0.0026$ [41].

In order to complete a calculation, we have to specify the detailed structure of the weak-current, and to calculate the nuclear wave functions. These will combine to produce the theoretical E_1^A , which will be compared to the experimental ones above.

III. χ PT WEAK CURRENTS IN THE NUCLEUS

The main difference between the current work and previous ones, is the physical origin of the currents. The last two decades of theoretical developments have provided us with an effective theory of QCD, in the form of χ PT [13, 14, 15, 16]. The χ PT Lagrangian is constructed by integrating out QCD degrees of freedom of the order of $\Lambda_\chi \sim 1$ GeV and higher. It retains all assumed symmetry principles, particularly the approximate chiral symmetry of the underlying theory. This $SU(2)_A \times SU(2)_V$ symmetry is based on the small up- and down-quarks masses (compared to the QCD breaking scale). The lack of parity doublets in the QCD scale is interpreted as an indication that this symmetry is spontaneously broken, with the pions as the Goldstone-Nambu bosons. Their finite, albeit small, mass is due to the finite quark masses, explicitly breaking the chiral symmetry.

Furthermore, the chiral Lagrangian can be organized in terms of a perturbative expansion in positive powers of Q/Λ_χ where Q is the generic momentum in the nuclear process, i.e. the β -decay or the pion mass [13, 14, 15]. The Chiral symmetry dictates the operator structure of each term of the effective Lagrangian, however not the coupling constants. A theoretical evaluation of these coefficients, or low-energy constants (LECs), is equivalent to solving QCD at low-energy, and it is not yet feasible to obtain them from lattice calculations because of computational limitations. Alternatively, these undetermined constants can be constrained by low-energy experiments.

As the chiral symmetry is a gauging of the electro-weak interaction, the weak currents are the Nöther currents of this symmetry. The weak axial current adopted in this work is the Nöther current derived from the axial symmetry of the chiral Lagrangian up to N^3 LO [27, 28]. At leading order (LO) this current consists of the standard single-nucleon part, which, as mentioned above, at low momentum transfer is proportional to the Gamow-Teller (GT) operator,

$$E_1^A|_{\text{LO}} = i g_A (6\pi)^{-1/2} \sum_{i=1}^A \sigma_i \tau_i^+, \quad (4)$$

where σ_i, τ_i^+ are spin and isospin-raising operators of the i th nucleon.

Corrections to the single-nucleon current appear at N^2 LO in the form of relativistic terms. It is easily verified [27] that the single nucleon current achieved in the χ PT formalism, is identical to that achieved in the standard nuclear physics approach (SNPA).

At N^3 LO, additional corrections appear in the form of axial MEC. While the relativistic corrections are negligible for the half life, the MEC have a substantial influence on this β -decay rate. This is a reflection of the fact that E_1^A is a chirally unprotected operator [29]. The MEC, to this order, include two topologies: a one-(charged)-pion exchange, and a contact term (that represents, for example, two-pion exchange or the exchange of heavier

mesons). In configuration space the one-pion exchange part of the axial MEC is given by:

$$\begin{aligned} -\frac{2Mf_\pi^2}{g_A} \hat{\mathcal{A}}_{1\pi}^{i,a}(r_{ij}) = & \mathcal{O}_P^{i,a} y_{1\Lambda}^\pi(r_{ij}) + \\ & + \hat{c}_3 (\mathcal{T}_\oplus^{i,a} - \mathcal{T}_\ominus^{i,a}) m_\pi^2 y_{2\Lambda}^\pi(r_{ij}) + \\ & + \frac{\hat{c}_3}{3} (\mathcal{O}_\oplus^{i,a} - \mathcal{O}_\ominus^{i,a}) m_\pi^2 y_{0\Lambda}^\pi(r_{ij}) - \\ & - (\hat{c}_4 + \frac{1}{4}) m_\pi^2 \left(\mathcal{T}_\otimes^{i,a} y_{2\Lambda}^\pi(r_{ij}) + \frac{2}{3} \mathcal{O}_\otimes^{i,a} y_{0\Lambda}^\pi(r_{ij}) \right). \end{aligned}$$

Where $f_\pi \approx 92.4$ MeV is the pion-decay constant, $M \approx 938.9$ MeV is the mass of the nucleon, $m_\pi \approx 139.57$ MeV is the charged-pion mass [1], and the low-energy constants $\hat{c}_3 = -3.66(8)$ and $\hat{c}_4 = 2.11(9)$ are calibrated in the π -N sector [30]. The operators used here are defined as,

$$\begin{aligned} \vec{\mathcal{O}}_P^a & \equiv -\frac{m_\pi}{4} (\vec{\tau}^{(1)} \times \vec{\tau}^{(2)})^a (\vec{P}_1 \vec{\sigma}^{(2)} \cdot \hat{r}_{12} + \vec{P}_2 \vec{\sigma}^{(1)} \cdot \hat{r}_{12}) \\ \mathcal{O}_\odot^{i,a} & \equiv (\vec{\tau}^{(1)} \odot \vec{\tau}^{(2)})^a (\vec{\sigma}^{(1)} \odot \vec{\sigma}^{(2)})^i \\ \mathcal{T}_\odot^{i,a} & \equiv \left(\hat{r}_{12}^i \hat{r}_{12}^j - \frac{\delta^{ij}}{3} \right) \mathcal{O}_\odot^{i,a}, \end{aligned}$$

and $\odot = \times, +, -$. In addition, the Yukawa-like functions are:

$$\begin{aligned} y_{0\Lambda}^\pi(r) & \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{r}} S_\Lambda^2(\vec{k}^2) \frac{1}{k^2 + m_\pi^2} \\ y_{1\Lambda}^\pi(r) & \equiv -\frac{\partial}{\partial r} y_{0\Lambda}^\pi(r) \\ y_{2\Lambda}^\pi(r) & \equiv \frac{1}{m_\pi^2} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} y_{0\Lambda}^\pi(r). \end{aligned}$$

S_Λ is a cutoff function, which we take as a Gaussian.

Apart from this, the MEC include a contact term, that has the form:

$$\frac{2Mf_\pi^2}{g_A} \hat{\mathcal{A}}_C^{i,a}(r_{ij}) = \hat{d}_r \mathcal{O}_\otimes^{i,a} \delta_\Lambda^{(3)}(\vec{r}_{ij}), \quad (5)$$

where the "smeared" delta function is

$$\delta_\Lambda^{(3)}(\vec{r}) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{r}} S_\Lambda^2(\vec{k}^2). \quad (6)$$

The LEC \hat{d}_r is the only LEC up to N^3 LO that cannot be calibrated in the single nucleon sector, as it originates in the contact interaction π -NN in the chiral Lagrangian. As a result, in order to determine \hat{d}_r , one has to resort to a larger nuclear system. We will use the triton half-life as an experimental datum to determine this LEC, Sec. V A.

IV. NUCLEAR WAVE FUNCTIONS

The difference between the one-body contribution to the ${}^6\text{He}$ - ${}^6\text{Li}$ GT matrix element and the experimental

value is of the order of few percent. A result which on the one hand is very satisfying, but on the other hand implies that numerical accuracy at a *per mil* level is required if we regard the ${}^6\text{He}$ β -decay as a test of the MEC model. In view of this required level of convergence we use the JISP16 potential [21] to model the interaction between the nucleons. The JISP16 NN potential utilizes the J-matrix inverse scattering technique to construct a soft nuclear potential, formulated in the harmonic oscillator basis, that by construction reproduces the NN phase shifts up to pion threshold and the binding energies of the light nuclei with $A \leq 4$.

We use the Hyperspherical-Harmonics (HH) expansion to solve the Schrödinger equation. The HH functions constitute a general basis for expanding the wave functions of an A -body system [31]. In the HH method, the translational invariant wave-function is written as

$$\Psi = \sum_{n[K]} C_{n[K]} R_n(\rho) \mathcal{Y}_{[K]}(\Omega, s_i, t_i) \quad (7)$$

where ρ is the hyperradius, and $R_n(\rho)$ are a complete set of basis functions. The hyperangle, Ω , is a set of $3A - 4$ angles, and $\mathcal{Y}_{[K]}(\Omega, s_i, t_i)$ are a complete set of antisymmetric basis functions in the Hilbert space of spin, isospin and hyperangles. The hyperradius ρ is symmetric under particle permutations since $\rho^2 = \frac{1}{2A} \sum_{i,j} (\mathbf{r}_i - \mathbf{r}_j)^2$. The functions $\mathcal{Y}_{[K]}(\Omega, s_i, t_i)$ are characterized by a set of quantum numbers $[K]$ [18, 19] and possess definite angular momentum, isospin, and parity quantum numbers. They are the eigenfunctions of the hyperspherical, or generalized angular momentum operator \hat{K}^2 , $\hat{K}^2 \mathcal{Y}_{[K]}(\Omega, s_i, t_i) = K(K + 3A - 5) \mathcal{Y}_{[K]}(\Omega, s_i, t_i)$. The details of our method are explained thoroughly in Ref. [32].

V. RESULTS

A. The triton β -decay - Calibration of \hat{d}_r

Our results for the ground state properties of the $A = 3$ nuclei, ${}^3\text{H}$ and ${}^3\text{He}$, are presented in table I. In the table, we present the energies, matter radii, and the leading order GT matrix element (see Eq. (4)) as a function of K_{max} , the limiting value of the hyperspherical angular momentum K in the HH expansion. As we are using the bare interaction our results are variational.

From the table it is evident that an excellent convergence is achieved for the $A = 3$ nuclei. Our results indicate that the JISP16 potential leads to an underbinding of about 80keV for the ${}^3\text{He}$ and 120keV for the triton. Comparing our results with the NCSM results of Shirokov *et al.* [21], we see a nice agreement with their variational results $[21]_V$ but a discrepancy of about 130keV with their effective interaction results $[21]_E$. It should be noted that the GT matrix element converges much faster than the matter radius. This property can be probably attributed to the fact that the GT is a medium-range

TABLE I: The JISP16 NN interaction ${}^3\text{He}$, ${}^3\text{H}$ binding energies, rms matter radius, and the leading order GT matrix element as a function of K_{max} .

K_{max}	${}^3\text{H}$		${}^3\text{He}$		GT _{LO}
	B.E.	radius	B.E.	radius	
4	8.094	1.632	7.364	1.653	1.6656
6	8.233	1.656	7.512	1.680	1.6620
8	8.319	1.677	7.604	1.704	1.6575
10	8.351	1.691	7.641	1.720	1.6547
12	8.360	1.697	7.651	1.727	1.6538
14	8.365	1.701	7.657	1.733	1.6530
16	8.367	1.704	7.660	1.736	1.6526
18	8.367	1.705	7.661	1.738	1.6524
$[21]_V$	8.354		7.648		
$[21]_E$	8.496(20)		7.797(17)		
Exp.	8.482		7.718		

operator, which is influenced by the asymptotic behavior of the wave function, described correctly using the hyperspherical functions. Comparing the JISP16 leading order GT matrix element with those of other potential models, see table II, we observe that the JISP16 potential model leads to an enhancement of the 1-body matrix element and it almost coincides with the experimental value. This property is found also for the UCOM potential, and might be a result of the minimization of the contribution of 3NF to the binding energy, which is in the essence of both these potentials. In general, one observes from the table that non-local potentials, such as Bonn or the N^3LO potentials, tend to predict a value for the GT matrix element which is closer to experiment than the local potentials.

TABLE II: The dependence of the triton β -decay leading order GT matrix-element on the potential model.

Potential model	GT _{LO}
AV18+3NF [33]	1.598(2)
Bonn+3NF [34]	1.621(2)
Nijm+3NF [35]	1.605(2)
N^3LO +3NF [39]	1.622(2)
UCOM [40]	1.65(1)
JISP16 [This work]	1.6524(2)
Expt.	1.656(3)

As explained in Sec. III, we use the triton half-life as an experimental input to determine the LEC \hat{d}_r . That is, we use the trinuclei wave functions to evaluate the matrix-element $|\langle {}^3\text{He} || E_1^A || {}^3\text{H} \rangle|$, of the E_1^A operator built from the χPT based weak-current, as a function of \hat{d}_r , for various high-energy-cutoff values. Using the experimentally derived value for this matrix element we get the following calibration for $\hat{d}_r(\Lambda_\chi)$:

$$\begin{aligned} \hat{d}_r(\Lambda_\chi = 500 \text{ MeV}) &= 0.583(27)_t(38)_{g_A} \\ \hat{d}_r(\Lambda_\chi = 600 \text{ MeV}) &= 0.625(25)_t(35)_{g_A} \\ \hat{d}_r(\Lambda_\chi = 800 \text{ MeV}) &= 0.673(23)_t(33)_{g_A}. \end{aligned} \quad (8)$$

The numbers in parenthesis denote uncertainties in the last digits. The first error is due to the uncertainty in the triton half-life, whereas the second one is due to uncertainty in g_A (the numerical error is negligible).

B. The ${}^6\text{He}$ - ${}^6\text{Li}$ Gamow-Teller matrix-element

Turning now to the $A = 6$ case, we present in table III our results for the ground state properties of the ${}^6\text{He}$, and ${}^6\text{Li}$ nuclei. As evident in the table, at the value $K_{max} = 14$, which corresponds to about $2-3 \cdot 10^6$ basis states, the binding energies of the 6-body nuclei are obtained with an accuracy of few hundreds keV.

TABLE III: The JISP16 NN interaction ${}^6\text{He}$, ${}^6\text{Li}$ binding energies, rms matter radii, and the leading order GT matrix element as a function of K_{max} .

K_{max}	${}^6\text{He}$		${}^6\text{Li}$		GT _{LO}
	B.E.	radius	B.E.	radius	
4	18.367	1.840	19.392	1.859	2.263
6	24.103	1.902	26.124	1.909	2.247
8	26.392	1.979	28.854	1.984	2.234
10	27.560	2.051	30.156	2.051	2.232
12	28.112	2.112	30.797	2.110	2.229
14	28.424	2.165	31.132	2.160	2.227
∞	28.70(13)		31.46(5)		2.225(2)
[21]	28.32(28)		31.00(31)		
Exp.	29.269	2.18	31.995	2.09	2.170

Taking a closer look at the table, we find that the binding energies exhibit an exponential convergence. Deploying this observation we extrapolate our results to the $K_{max} \rightarrow \infty$ limit, using the formula $E(K_{max}) = E_\infty + Ae^{-\alpha K_{max}}$. Fitting the parameters E_∞, A, α to the entries of table III in the range $K_{max} \geq 6, 8, 10$ we find a rather stable value for E_∞ with variance of about 50keV for ${}^6\text{Li}$ and 130keV for ${}^6\text{He}$. The resulting binding energies are 28.70MeV for ${}^6\text{He}$ and 31.46MeV for ${}^6\text{Li}$. While these results are roughly 550keV below the experimental values, the difference $\Delta E = 2.76\text{MeV}$ between the binding energies of the two nuclei differs by merely 34keV from the experimental value, $\Delta E = 2.726\text{MeV}$. In the last column of table III we present our ${}^6\text{He}$ - ${}^6\text{Li}$ leading order GT transition matrix element, i.e. at the 1-body level. It can be seen that the convergence pattern of the matrix element is not regular. Extrapolating its value using the expression $\text{GT}(K_{max}) = \text{GT}_\infty + \text{Be}^{-\beta K_{max}}$ for $K_{max} \geq 0$, we get $\text{GT}_\infty = 2.225(2)$. The fits of the extrapolation formulae to the calculated values are presented in Fig. 1 for the binding energies and in Fig. 2 for the GT matrix-element.

The value $\text{GT} = 2.225(2)$ we obtained for the JISP16 potential, is in accordance with the values $\text{GT} = 2.28$ for AV8'/TM'(99) and $\text{GT} = 2.30$ for AV8' obtained by Navratil and Ormand [36], $\text{GT} = 2.28$ for the N^3LO NN-force of Navratil and Caurier [37], $\text{GT} = 2.25$ for AV18/UIX of Schiavilla and Wiringa [11], and $\text{GT} =$

$2.16-2.21$ for AV18/IL2 by Pervin *et al.* [12]. Moreover, it can be seen that our accuracy in estimating the GT matrix element is at the level of *per mil*. Such an accuracy enables us to disentangle numerics from physics and validates the use of the ${}^6\text{He}$ β -decay as a testing ground for an axial MEC model.

Incorporating the χPT based contributions to the weak-current we can finally calculate the full ${}^6\text{He}$ - ${}^6\text{Li}$ GT matrix-element at the N^3LO level. In table IV, we present the transition matrix-elements as a function of K_{max} and the cutoff Λ_χ . The appropriate values of \hat{d}_r are taken from Eq. (8). Two important observations can be drawn from the table, (i) the numerical accuracy of the calculated GT matrix-element is few per mil, and (ii) there is only a very weak dependence on the cutoff Λ_χ , which is of the same order of magnitude. The second observation implies that there is no need to refine our calculation, and moreover, the contribution of higher order χPT corrections to the weak-current are negligible.

TABLE IV: The dependence of the full (1-body+2-body) ${}^6\text{He}$ - ${}^6\text{Li}$ GT matrix-element on K_{max} as a function of the cutoff Λ_χ , at the N^3LO level.

K_{max}	$\Lambda_\chi = 500\text{MeV}$	$\Lambda_\chi = 600\text{MeV}$	$\Lambda_\chi = 800\text{MeV}$
4	2.1870	2.1798	2.1703
6	2.1850	2.1805	2.1746
8	2.1868	2.1850	2.1826
10	2.1937	2.1932	2.1927
12	2.1951	2.1952	2.1955
14	2.1970	2.1975	2.1983

Summarizing, the predicted GT of ${}^6\text{He}$ is:

$$|\text{GT}({}^6\text{He})|_{\text{theo}} = 2.198(1)_\Lambda(2)_\text{N}(4)_\text{t}(5)_{g_A} = 2.198 \pm 0.007 \quad (9)$$

The first error is the cutoff variation dependence, the second is numerical, the third is due to uncertainties in the triton half-life, and the last is due to uncertainties in g_A . This should be compared to the experimental matrix-element $|\text{GT}({}^6\text{He})|_{\text{expt}} = 2.161 \pm 0.005$. Thus, the theory overpredicts GT by about 1.7%.

VI. DISCUSSION

The use of phenomenologically based potential, JISP, combined with a χPT based MEC, is an inconsistency inherent to our calculation. Clearly, the chiral Lagrangian can be used to derive the nuclear forces as well. This inconsistency, however, allows us to overcome limited computational resources, as well as theoretical difficulties (the N^3LO nuclear potential has not been fully developed yet), and to accomplish the task of a microscopic calculation of a six-body problem. The use of a hybrid approach, sometimes coined EFT*, has had great success in the literature [27, 28, 38]. In all these checks, the phenomenological nuclear forces included realistic potentials, the AV18+UIX force model. This potential, though

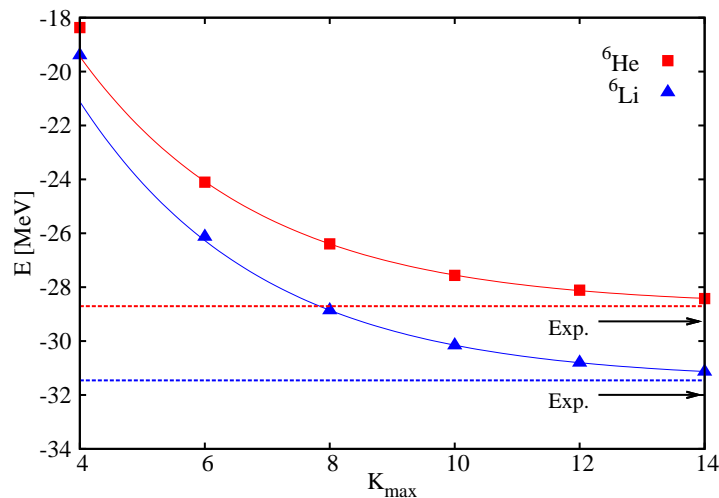


FIG. 1: (Color online) The convergence of the binding energies of the 6-body nuclei, ${}^6\text{Li}$ and ${}^6\text{He}$. The continuous lines are the fits $E(K_{\max}) = E_{\infty} + Ae^{-\alpha K_{\max}}$. The dashed lines are the extrapolated values E_{∞} . The experimental values are marked with black arrows.

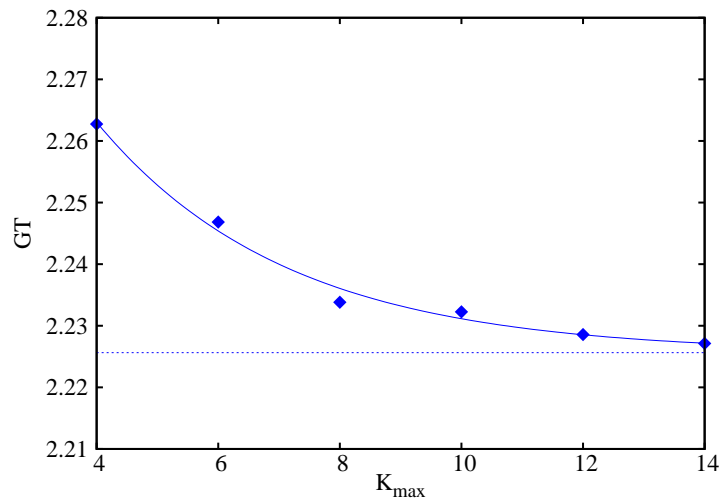


FIG. 2: (Color online) The convergence of the GT matrix element for the ${}^6\text{He}$ - ${}^6\text{Li}$ β -decay. The continuous line is the fit $\text{GT}(K_{\max}) = \text{GT}_{\infty} + Be^{-\beta K_{\max}}$, the dashed line is the extrapolated value GT_{∞} .

different than the χPT force models in the short-range character of the force, has the correct long-range behavior, due to the pion exchange. The JISP potential, however, is different in this respect, as it is built in an *ab-exitu* approach, and does not have an asymptotic long pion behavior, thus not consistent with chiral symmetry even at long-distances. In addition, the JISP potential does not include a three-body force.

It is hard to estimate the effect of these approximations. However, in a recent work [39], the triton β -decay process was calculated using force model and current derived consistently from the same χPT N^3LO Lagrangian. One of the conclusions of this work has been that the short-range correlations of the force and the short-range

correlations of the weak current are not correlated, thus the effect of the three-body-force is negligible for GT-type operators. In addition, the JISP potential successfully reproduces nucleon-nucleon scattering data, and the binding energies of $A < 16$ mass nuclei. However, the most convincing reason to believe the stability of the current results, is the minimal dependence of the half-life in the cutoff.

We thus believe that even in the current calculation, the effect of the approximation will not change qualitatively the results, and the effect of the MEC. The qualitative difference originates in the different structure of the SNPA and χPT based MEC.

A careful analysis of the difference between the MEC

originating in χ PT and those used in SNPA, has been accomplished by Park *et al* [27]. They have shown that one-pion exchange term exists in both models. Of particular importance is the part of this term in the SNPA based MEC that represents the exchange of a pion due to a delta excitation of the nucleon, which is found to correspond roughly to the \hat{c}_3 term in the χ PT based MEC. The coupling constant of this term $g_{\pi N\Delta}$ has been fixed by Schiavilla and Wiringa [11], so that the theory would reproduce the triton half-life.

However, differences between the approaches arise in their short range character. In the SNPA approach, these correspond to the exchange of a ρ -meson. Such a term does not exist in the χ PT approach as it arises only at N^5 LO [27]. Moreover, a contact interaction of the form of Eq. (6) does not appear in the SNPA approach. It is this contact interaction that creates the qualitative difference between the current work and that of Schiavilla and Wiringa [11].

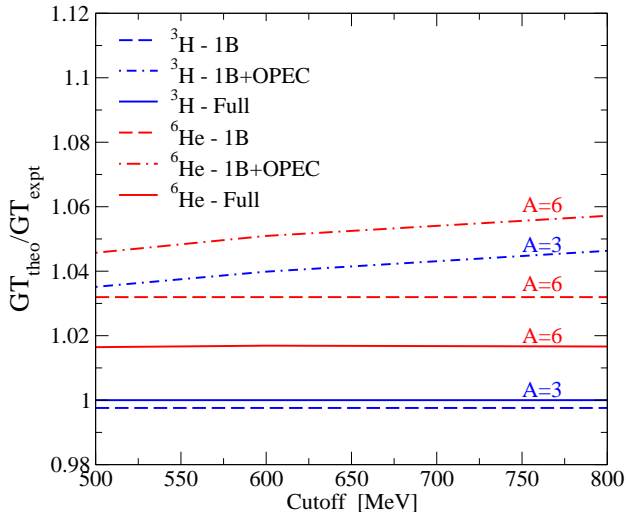


FIG. 3: (Color online) Relative contributions to the theoretical GT matrix elements as a function of the EFT cutoff. All the results are normalized to the empirical values. The blue lines indicated by $A = 3$ correspond to the ${}^3\text{H}$ - ${}^3\text{He}$ β -decay. The red lines indicated by $A = 6$ correspond to the ${}^6\text{He}$ - ${}^6\text{Li}$ case. Dashed lines correspond to the 1-body impulse approximation (1B). Dashed-dotted lines correspond to 1-body plus one-pion exchange current (OPEC). Continuous lines correspond to full calculation (note that in the case of ${}^3\text{H}$ this is calibrated to give exactly the experimental value).

In order to acknowledge that, we plot in Fig. 3 the relative contribution of each the terms, i.e. one-body, one-pion-exchange and full calculation, to the GT matrix element. One first recognizes that the one-pion contribution to the matrix element has a positive sign in both ${}^3\text{H}$ and ${}^6\text{He}$, and that the contact interaction has a negative contribution to the matrix element. In the case of ${}^3\text{H}$ this is only a partial cancellation, as it is calibrated to increase the 1-body matrix element and to bring the cal-

culation into the experimental value. In view of the fact that the one-body calculation in the case of the JISP potential almost exhausts the total GT strength, one might suggest that the negative sign of the contact interaction, as well as the partial cancellation is an artifact of the potential. However, the same partial cancellation is found also in a consistent N^3 LO calculation of ${}^3\text{H}$ decay, thus it is not a result of the use of the JISP potential [39].

In contrast to ${}^3\text{H}$, when examining the case of ${}^6\text{He}$, one observes that the negative contribution of the contact term is bigger (in absolute value) than the one-pion-exchange contribution, thus leading to a total negative contribution of the MEC. This negative contribution is needed as the single-nucleon GT is bigger than the experimental GT.

Recalling the fact that the SNPA approach does not contain a contact interaction, we understand the origin of the positive contribution of the MEC in that approach, which increases the difference between the calculated and measured decay-rates.

VII. SUMMARY AND CONCLUSIONS

In this work we have used the ${}^6\text{He}$ β -decay as a testing ground for the nuclear weak-current derived from χ PT. A precondition for such a task is an accurate evaluation of the ${}^6\text{He}$ - ${}^6\text{Li}$ weak transition matrix-element at the per mil level. To this end we have used the soft NN potential JISP16 to describe the nuclear dynamics and the HH expansion method to solve the Schrödinger equation. The weak interaction in the nucleus is completely determined by fixing the short range behavior of the scattering operator to reproduce the experimental ${}^3\text{H}$ half-life, resulting in a parameter-free prediction of the ${}^6\text{He}$ β -decay rate.

We have found that at the 1-body, impulse approximation, level the ${}^6\text{He}$ - ${}^6\text{Li}$ GT matrix-element is over predicted by roughly 3%. This observation for the JISP16 potential is in agreement with previous findings for other potential models. Adding 2-body, meson-exchange, currents derived within χ PT, we have found that in contrast with the previous work of Schiavilla and Wiringa [11], the 2-body MEC contribution to the ${}^6\text{He}$ - ${}^6\text{Li}$ transition matrix element is negative. We argue that this difference originates in the different short-range character of the MEC derived in the two approaches. We find that both for ${}^3\text{H}$ and ${}^6\text{He}$, there is a sign difference between the positive contribution of the long-range one-pion-exchange current, and the negative contribution of the contact interaction in χ PT, representing higher degrees of freedom which were integrated out in the development of the effective theory. In the case of the 6-body transition, however, the contact interaction has a bigger value than the one-pion exchange contribution. Thus, it provides the origin to the sign difference between the MEC contribution in ${}^3\text{H}$ and ${}^6\text{He}$. This contact interaction does not exist in the standard nuclear physics approach, adopted by Schiavilla and Wiringa. Therefore, the reconciliation between

the theoretical and the experimental ${}^6\text{He}$ half-life is due to the use of the χPT formalism.

Our calculation points to an agreement at the level of about 1.7% between the measured and calculated GT matrix elements. This result should be contrasted with the difference of 5.4% obtained by Schiavilla *et al.* [11]. More importantly, it shows that dominant contributions that arise naturally in the χPT formalism, and do not appear in the standard nuclear physics approach, are essential to a successful prediction of this weak observable. In order to pin-point this argument, a use of a consistent approach, in which both the weak currents and the nuclear forces are derived from the same microscopic theory, is called for.

The agreement between the calculated and measured

decay rates of ${}^6\text{He}$ indicates that there is no signature in this observable for an additional suppression of the axial constant. It appears that all the needed suppression originates in correlations between nucleons in the nucleus, revealing itself in the form of exchange currents.

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